Conclusions. A numerical method is offered for solving the system of nonlinear equations arising from the algebraic approximation of equations of radiation and convection energy transport. The classic Newton–Raphson scheme is proposed as a foundation of the method. The use of the method of conjugate gradients with preliminary conditioning and symmetrization of the matrix of the linearized system results in considerable improvement in the computer memory and computational time as compared with the Newton–Raphson scheme in systems with the number of variables exceeding 100.

An application of preconditioning on the basis of the substitution of variables (12) allows one to solve effectively the problems of complicated heat exchange in systems with pairs of "strongly interacting" zones, such as thin screens.

NOTATION

T, temperature vector; S, vector of external sources in the zones; R, matrix of coefficients of radiation heat exchange; A, convective matrix, B, matrix of linearized system; σ, Stefan–Boltzmann constant; λ, coefficient of thermal conductivity; h, plate thickness or thickness of the pipe wall; α, coefficient of absorption of heat carrier; W, water equivalent of radiating gas; T'_g, temperature of heat carrier at the input of the unit.

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SOLITARY STRESS WAVES IN A NONLINEAR THERMOELASTIC MEDIUM

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The propagation process of solitary stress waves in a medium with five thermoelastic characteristics is investigated within the one-dimensional statement. Existence conditions and geometric characteristics of solitary waves are obtained, and restrictions are found for the elastic and thermal constants.

1. Statement of the Problem. Propagation of one-dimensional waves in a thermoelastic medium in the absence of heat sources and sinks is described within five-constant nonlinear thermoelastic theory by the system of equations [1-3]:

$$\varepsilon = e + \frac{1}{2} e^2, \tag{1}$$

$$\sigma = c_1 e + c_2 e^2 - T, \tag{2}$$

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$$\frac{\partial^2\sigma}{\partial x^2} = \rho \frac{\partial^2 e}{\partial t^2}, \tag{3}$$

$$\lambda_0 \frac{\partial^2\theta}{\partial x^2} - c_\varepsilon \frac{\partial\theta}{\partial t} - T \frac{\partial e}{\partial t} = 0, \tag{4}$$

where

$$c_1 = K + \frac{4}{3}\mu; \quad c_2 = \frac{3}{2}\left(K + \frac{4}{3}\mu\right) + A + C + 3B;$$

$$e = \frac{\partial u}{\partial x}; \; T = \nu\theta.$$

Based on Eqs. (1)-(4), we consider the following problem: Find the shape of nonlinear dependences $\theta = \theta(\sigma)$, $\nu = \nu(\sigma)$ for which the stress waves have the nature of solitons. With this purpose in mind, we supplement the system (1)-(4) by equations characterizing solitary waves [4, 5]. As a result we reach the following system of differential equations [6]:

$$2v + c - 4k^2 = \left(\frac{\lambda_3}{g} + \frac{dv}{d\sigma}\right)\sigma, \tag{5}$$

$$\frac{dv}{d\sigma} g = [2v^3 + cv^2 + dv + b]^{1/2}, \tag{6}$$

$$h = -cg, \tag{7}$$

$$\frac{de}{d\sigma} = \frac{1}{\rho c^2} - \frac{m}{g}, \tag{8}$$

$$\lambda_0 \frac{d\theta}{d\sigma} + \frac{cc_\varepsilon\theta}{g} = n - \frac{c}{g}\left[\int_0^\sigma e\,d\sigma + \frac{c_1}{2}e^2 + \frac{c_2}{3}e^3 - e\sigma\right], \tag{9}$$

where $g(\sigma) = \partial\sigma/\partial x$; $h(\sigma) = \partial\sigma/\partial t$; λ_3, b, c, d, m, n are constants.

<u>2. The Case ν = const, m = n = 0.</u> We introduce the new variable

$$v = 2y - \frac{c}{6}. \tag{10}$$

Substituting (10) into (5), (6), we obtain:

$$4\left(y + \frac{c}{6} - k^2\right) = \left(\frac{\lambda_3}{g} + 2\frac{dy}{d\sigma}\right)\sigma, \tag{11}$$

$$\frac{dy}{d\sigma} g = (4y^3 - g_2 y - g_3)^{1/2}, \tag{12}$$

where

$$g_2 = \frac{c^2}{12} - \frac{d}{2}, \; g_3 = \frac{cd}{24} - \frac{c^3}{6^3} - \frac{b}{4}.$$

It is assumed that e_1, e_2, e_3 are real roots of the equation

$$4y^3 - g_2 y - g_3 = 0.$$

Equation (12) can then be rewritten as:

$$\frac{dy}{d\sigma} g = 2\,[(y - e_1)(y - e_2)(y - e_3)]^{1/2}, \tag{13}$$

where $e_1 + e_2 + e_3 = 0$.

If $v = 2e_i - (c/6)$, $i = \overline{1, 3}$, it then follows from (11), (13) that

$$g = -\lambda\sigma, \ \lambda = \frac{\lambda_3}{4\left(k^2 - \frac{c}{6} - e_i\right)} > 0. \tag{14}$$

Based on (7), (14) we have

$$\sigma = \beta \exp(-\lambda(x - ct)), \ 0 \leqslant \sigma \leqslant \beta \ \text{ for } x - ct > 0, \tag{15}$$

where β is a constant of integration.

Integrating Eq. (8) with account of (14), we obtain

$$e = \frac{1}{\rho c^2}\sigma + \frac{m}{\lambda}\ln\sigma. \tag{16}$$

If m ≠ 0, then e = −∞ when σ = 0. We therefore choose m = 0, and then

$$e = \frac{1}{\rho c^2}\sigma. \tag{17}$$

By (17) it follows from (1), (2), (9) that:

$$\varepsilon = \frac{1}{\rho c^2}\sigma + \frac{1}{2\rho^2 c^4}\sigma^2, \tag{18}$$

$$T = \left(\frac{c_1}{\rho c^2} - 1\right)\sigma + \frac{c_2}{\rho^2 c^4}\sigma^2; \tag{19}$$

$$\theta = a_1 \sigma^{cc_\varepsilon/\lambda\lambda_0} + a_2\sigma^2 + a_3\sigma^3, \tag{20}$$

where

$$a_3 = \frac{c_2}{3\rho^3 c^6\left(\frac{3\lambda\lambda_0}{c} - c_\varepsilon\right)}; \ a_2 = \frac{1}{2\rho c^2}\,\frac{\frac{c_1}{\rho c^2} - 1}{\left(\frac{2\lambda\lambda_0}{c} - c_\varepsilon\right)};$$

and a_1 is a constant of integration.

For T and θ to be positive quantities for all σ it is required that:

$$\frac{\lambda\lambda_0}{c} = c_\varepsilon, \ \frac{c_1}{\rho} > c^2. \tag{21}$$

Since T = νθ, the heat conduction coefficient ν is determined as follows:

$$\nu = \frac{\frac{c_2}{\rho^2 c^4}\sigma + \frac{c_1}{\rho c^2} - 1}{a_3\sigma^2 + a_2\sigma + a_1}. \tag{22}$$

If

$$a_1 = \frac{1}{2c_2 c_\varepsilon}\rho c^2\left(\frac{c_1}{\rho c^2} - 1\right)^2, \tag{23}$$

then ν is a monotonically decreasing function of σ. Consequently,

$$\frac{\frac{c_2\beta}{\rho^2 c^4} + \frac{c_1}{\rho c^2} - 1}{a_3\beta^2 + a_2\beta + a_1} \leqslant \nu \leqslant \frac{2c_2 c_\varepsilon}{c_1 - \rho c^2} \tag{24}$$

Let the source, creating a solitary wave, be located at the origin of coordinates. It follows from (7), (14), (15) that

$$\left.\begin{aligned}\sigma &= \beta = \sigma_0,\\ \frac{\partial\sigma}{\partial t} &= c\lambda\beta = h_0\end{aligned}\right\} \text{ for } x = 0,\ t = 0. \tag{25}$$

For known σ_0, h_0 we have by means of (21), (24)

$$c^2 = \frac{\lambda_0 h_0}{c_2\sigma_0}. \tag{26}$$

The quantities a_1, β, c^2 in inequality (24) are determined by Eqs. (23), (25), (26).

3. The Case $e_1 = e_2$. Eliminating g from (11), (13), we obtain

$$\frac{dy}{2\left(y + \frac{c}{6} - k^2\right)} + \frac{\lambda_3 dy}{8\left(y + \frac{c}{6} - k^2\right)(y - e_2)(y - e_3)^{1/2}} = \frac{d\sigma}{\sigma}. \tag{27}$$

For $y + c/6 - k^2 < 0$ the solution of Eq. (27) is

$$\left(y + \frac{c}{6} - k^2\right)\exp(\lambda_3 I_3) = -R^2\sigma^2, \tag{28}$$

where R^2 is a constant of integration, and

$$I_3 = \frac{1}{4}\int \frac{dy}{\left(y + \frac{c}{6} - k^2\right)(y - e_2)(y - e_3)^{1/2}}. \tag{29}$$

We put

$$z^2 = y - e_3, \tag{30}$$

$$p^2 = k^2 - \frac{c}{6} - e_3,\ q^2 = e_2 - e_3. \tag{31}$$

We then have

$$\sigma = \frac{1}{R}(p^2 - z^2)^{1/2}\left(\frac{z - q}{z + q}\right)^{\frac{\lambda_3}{8q(q^2 - p^2)}}\left(\frac{z + p}{z - p}\right)^{\frac{\lambda_3}{8p(q^2 - p^2)}}. \tag{32}$$

Passing in the right hand side of Eq. (32) to the limit $p \to q$, we obtain

$$\sigma = \frac{1}{R}(q + z)\exp\left(\frac{qz}{q^2 - z^2}\right) \text{ for } \lambda_3 = 8q^3,\ |z| < q. \tag{33}$$

For $p > q$ it follows from (32) that

$$\sigma = \frac{1}{R}(p - z)\left(\frac{q + z}{q - z}\right)^{\frac{p}{2q}} \text{ for } \lambda_3 = 4p(p^2 - q^2),\ |z| < q. \tag{34}$$

By means of (13) we have

$$z = -q\,\mathrm{th}\,q\xi,\ \xi = x - ct. \tag{35}$$

Substituting (35) into (33), (34), we obtain

$$\sigma = \frac{q}{R}(1 - \mathrm{th}\,q\xi)\exp\left(-\frac{1}{2}\ \mathrm{sh}\,2q\xi\right), \tag{36}$$

$$\sigma = \frac{1}{R}(p + q\,\mathrm{th}\,q\xi)\left(\frac{1 - \mathrm{th}\,q\xi}{1 + \mathrm{th}\,q\xi}\right)^{\frac{p}{2q}}. \tag{37}$$

Equations (36), (37) provide finite-amplitude solitons when $\xi = x - ct \geq 0$. These solitons, however, cannot exist in the medium under consideration, since the condition of finite wave energy is violated ($e = +\infty$ when $\sigma = 0$). We turn attention to the case

$$\sigma = \frac{1}{R}(p - z)\left(\frac{z+q}{z-q}\right)^{\frac{p}{2q}}, \quad |z| > q. \tag{38}$$

In that case Eq. (13) provides the solution

$$z = q \operatorname{cth} q(\xi_1 - \xi). \tag{39}$$

For σ to be larger than zero, on the basis of (38), (39), ξ_1 and ξ must satisfy the conditions

$$0 \leqslant \xi = x - ct \leqslant \xi_2, \quad q \operatorname{cth} q(\xi_1 - \xi_2) = p. \tag{40}$$

Passing in the right hand sides of Eqs. (38), (39) to the limit $q \to 0$, we obtain

$$\sigma = \frac{1}{R}(p - z)e^{\frac{p}{z}}, \tag{41}$$

$$z = \frac{1}{\xi_1 - \xi}, \quad \frac{1}{\xi_1} \leqslant z \leqslant \frac{1}{\xi_1 - \xi_2}. \tag{42}$$

Taking (41) into account, the initial data at the source are

$$\left.\begin{aligned} \sigma &= \frac{1}{R}\left(p - \frac{1}{\xi_1}\right)\exp(p\xi_1) = \sigma_0, \\ \frac{\partial\sigma}{\partial t} &= \frac{c}{R}\left(\frac{1}{\xi_1^2} - \frac{p}{\xi_1} + p^2\right)\exp(p\xi_1) = h_0 \end{aligned}\right\} \text{for } x = 0,\ t = 0. \tag{43}$$

Besides, form (41) we have

$$g = -\frac{1}{R}(z^2 - pz + p^2)\exp(p/z). \tag{44}$$

Substituting (44) into (8) and integrating over z, we obtain

$$e = (p - z)\left(\frac{\exp(p/z)}{R\rho c^2} + \frac{m}{pz}\right), \quad e = 0 \text{ for } \sigma = 0. \tag{45}$$

As an example of determining the temperature consider the case m = n = 0. Then

$$e = \frac{1}{\rho c^2}\sigma = \frac{1}{R\rho c^2}(p - z)\exp\left(\frac{p}{z}\right). \tag{46}$$

On the basis of (2), (9), (46) we have

$$T = \frac{1}{R}\left(\frac{c_1}{\rho c^2} - 1\right)(p - z)\exp\left(\frac{p}{z}\right) + \frac{c_2}{R^2\rho^2c^4}(p - z)^2\exp\left(\frac{2p}{z}\right), \tag{47}$$

$$\frac{d\theta}{d\sigma} + \frac{cc_\varepsilon}{\lambda_0 g}\theta = -\frac{c}{\lambda_0 g}\left[\frac{1}{2\rho c^2}\left(\frac{c_1}{\rho c^2} - 1\right)\sigma^2 + \frac{c_2}{3\rho^3c^6}\sigma^3\right], \tag{48}$$

$$\theta = \frac{-1}{\varkappa}\int_0^\sigma \frac{c\varkappa}{\lambda_0 g}\left[\frac{1}{2\rho c^2}\left(\frac{c_1}{\rho c^2} - 1\right)\sigma^2 + \frac{c_2}{3\rho^3c^6}\sigma^3\right]\sigma, \tag{49}$$

where

$$\varkappa = \exp(-p_1 w); \quad p_1 = \frac{cc_\varepsilon}{\lambda_0}; \quad w = \frac{1}{z}. \tag{50}$$

With account of (44) we hence find that if $c_1/\rho > c^2$, then

$$T > 0, \quad \theta > 0, \quad \frac{d\theta}{d\sigma} > 0 \quad \forall\sigma. \tag{51}$$

When $cc_\varepsilon/\lambda_0 = p$, by means of (41), (44), Eq. (49) can be specified to the form:

$$\theta=\frac{c}{\lambda_0}\left\{\frac{1}{2R^2\rho c^2}\left(\frac{c_1}{\rho c^2}-1\right)\left[\left(p-\frac{1}{w}\right)\exp(2pw)-p\exp(pw)\times\right.\right.$$
$$\left.\times\int\limits_{1/p}^{w}\frac{\exp(pw)}{w}dw\right]+\frac{c_2}{6R^3\rho^3c^6}\left[\left(p^2+\frac{1}{w^2}-\frac{4p}{w}\right)\exp(3pw)+\right.$$
$$\left.\left.+2p^2\exp(2+pw)+2p^2\exp(pw)\int\limits_{1/p}^{w}\frac{\exp(2pw)}{w}dw\right]\right\}, \tag{52}$$

$\theta = 0$ for $w = 1/p$ or $\sigma = 0$.

Knowing σ_0, h_0, the constants c, p, ξ_1, ξ_2 are determined by Eqs. (40), (43), and (50). It follows from (47) and (52) that the heat conduction coefficient is $\nu = T/\theta$.

Similar calculations can be performed for the case $m \neq 0$.

NOTATION

Here μ, K denote second order elastic constants; μ is the shift modulus, K is the bulk modulus; A, B, C are the Landau third order elastic constants; λ_0, c_ε are thermal constants; θ is temperature; ν is the heat conduction coefficient; ε is the deformation; σ is the stress; u is the displacement; and ρ is the density of the medium.

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